Page

GENERAL

A	Area, m^2
Ar	Archimedes number = $\dfrac{Dp^3(\rho_s - \rho_g)\rho_g g}{\mu_g^2}$, dimensionless
B	Constant in Martin's model, dimensionless (= 2.6)
C_{pg}	Specific heat of fluidizing gas at constant pressure, J/kg K
C_s	Specific heat of solid particle, J/kg K
D	Diameter, m
e	emissivity, dimensionless
e_{eff}	effective bed-to-surface emissivity, dimensionless
G	Superficial mass velocity of fluidizing gas = $u\rho_g$, kg/m^2s
g	Acceleration due to gravity, m/s^2
h	Heat transfer coefficient, W/m^2 K
h_{cond}	Conductive component of heat transfer coefficient between wall and bed, W/m^2 K
h_{conv}	Convective component of heat transfer coefficient between bed and wall, W/m^2 K
h_{rad}	Radiative component of heat transfer coefficient between wall and bed, W/m^2 K
h_{st}	Heat transfer coefficient for a single tube in bed, W/m^2 K
h_w	Heat transfer coeffcient between bed and wall, W/m^2 K
$h_{w,max}$	Maximum value of h_w
k	Thermal conductivity, W/m K
M	Molecular weight
Nu	Nusselt number = h D/k, dimensionless
Nu_s	$= h_w D_s/k_g$
$Nu_{s,max}$	$= h_{w,max} D_s/k_g$
Nu_t	$= h_w D_t/k_g$
$Nu_{t,max}$	$= h_{w,max} D_t/k_g$
P	Pitch of tube bundle (center-to-center distance of adjacent tubes), m
Pr_g	Prandtl number of fluidizing gas = $C_{pg}\mu_g/k_g$, dimensionless
P_H	Horizontal pitch of tube bundle, m
P_V	Vertical pitch of tube bundle, m
p	Absolute pressure, N/m^2
q	Heat flux, W/m^2
R	Universal gas constant, J/kg-mole K
Re_{opt}	Optimum Reynolds number, i.e., Reynolds number at which $h_{w,max}$ occurs = $G_{opt} D_s/\mu_g$, dimensionless
T	Temperature, K
u	Superficial gas velocity (i.e. volumetric flow rate/ $(\pi D_b^2/4)$), m/s

Greek

ε	Bed voidage (also called void fraction), dimensionless
μ	Dynamic viscosity of gas, Ns/m^2
ρ	Density, kg/m^3
σ	Stefan-Boltzman constant = 5.792×10^{-8} $W/m^2 K^4$

Subscripts

b	of bed
g	of gas
mf	minimum fluidization
opt	optimum, i.e. corresponding to maximum in u - h_w curve
s	of solid particle
t	of tube, cylinder, or sphere
w	of wall

1.1 Grace, J. R., "Fluidized-Bed Heat Transfer," Section 8-2 in *Handbook of Multiphase Systems*, ed. G. Hetsroni, Hemisphere, Washington, DC, 1982.

1.2 Ainstein, V. G, "An Investigation of Heat Transfer Processes between Fluidized Beds and Single Tubes Submerged in the Beds," Kandi Dissertation, M. I. T. Kh. T. im. Lomonosova, Moscow, 1959. Quoted on page 270 in Zabrodsky [1.28].

1.3 Tode, O. M., in "Applications of Fluidized Beds in the Chemical Industries," part II, pp. 4-27. Quoted by Gelperin and Einstein [1.21].

1.4 Abubakr, Y. M., Bergaugnou,.M. A., Tarasuk, T. D., and Sullivan, J. L., "Local Heat Transfer Coefficients Around a Horizontal Tube in a Shallow Fluidized Bed," *J. Powder & Bulk Solids Technology*, vol. 4, pp. 11-18, 1980.

1.5 Baskakov, A. P., "Heat Transfer in Fluidized Beds: Radiative Heat Transfer in Fluidized Beds," Chapter 13B in *Fluidization*, 2nd edition, ed. J. F. Davidson, *et al*, Academic Press, London, 1985.

1.6 Xavier, A. M., and Davidson, J. F., "Heat Transfer in Fluidized Beds: Convective Heat Transfer in Fluidized Beds," Chapter 13A in *Fluidization*, ed. J. F. Davidson, *et al*, Acadamic Press, London, 1985.

1.7 Grewal, N. S., "Heat Transfer Between Tubes and Gas-Solid Fluid Beds," Chapter 18 in *Handbook of Heat and Mass Transfer Operations*, vol. 1, ed. N. P. Cheremisinoff, Gulf Publishing, Houston, 1986.

1.8 Bansal, R. K., Kadaba, P. V., and Desai, P. V., "Heat Transfer from Horizontal Tubes in Gas Fluidized Beds," ASME paper no. 80-HT-115, 1980.

1.9 Baskakov, A. P., Berg, B. V., Vitt, O. K., Fillippovsky, N. F., Kirakosyan, V. A., Goldobin, J. M.,and Maskaev, V. K., "Heat Transfer to Objects Immersed in Fluidized Beds," *Powder Technology*, vol. 8, pp. 273-282, 1973.

1.10 Baerg, A., Klassen, J., and Gishler, P. E., "Heat Transfer in a Fluidized Solids Bed," *Can. J. Research*, F28, pp. 287-307, 1950.

1.11 Bartel, W. J., Genetti, W. E., and Grimmet, E. S., "Heat Transfer from a Horizontal Discontinuous Finned Tube in a Fluidized Bed," *AIChE Symp. Ser.*, vol. 67, no. 116, 1971.

1.12 Bordulya, V. A., Ganzha, V. L., Zheltov, A. I., Upadhyay, S. N., Saxena, S. C., "Heat Transfer Between Gas-Solid Fluidized Beds and Horizontal Tube Bundles," *Letters in Heat Mass Transfer*, vol. 7, pp. 83-95, 1980.

1.13 Zabrodsky, S. S., Epanov, Yu. G., *et al*, "Heat Transfer in a Large-Particle Fluidized Bed with Immersed In-Line and Staggered Bundles of Horizontal Smooth Tubes," *Int. J. Heat Mass Transfer*, vol. 24, no. 4, pp. 571-579, 1981.

1.14 Ku, A. C., Kuwata, M., and Staub, F. W., "Heat Transfer to Horizontal Tube Banks in a Turbulent Fluidized Bed of Large Particles," *AIChE Symp. Ser.*, vol. 77, no. 208, pp. 359-367, 1981.

1.15 Golan, L. P., and Cherrington, D. C., "Heat Transfer and Unit Response to a Large Fluidized Bed Combuster," *AIChE Symp. Ser.*, vol. 77, no. 208, pp. 374-383, 1981.

1.16 Turton, R., Colakyon, M., and Levenspiel, O., "Heat Transfer from Fluidized Bed to Immersed Fine Wire," *Powder Technology*, vol. 53, No. 3, pp. 195-203, Dec. 15, 1987.

1.17 Botterill, J. S. M., "Fluid-Bed Heat Transfer," Acadamic Press, New York, 1975.

1.18 Baerns, M., "Fluidization of Fine Particles," *Proc. Int. Symp. Fluidization*, June 6-9, Eidhoven, p. 403, Netherland University Press, 1967.

1.19 Pitts, D. R., Figliola, R. S., and Hamlyn, K. M., "Heat Transfer between a Horizontal Cylinder and a Mixed Particle Size Fluidized Bed," *J. Heat Transfer*, vol. 104, pp. 563-565, 1982.

1.20 Goshayeshi, A., Welty, J. R., *et al*, "Local Heat Transfer Coefficients for Horizontal Tube Arrays in High-Tempertaure Large-Particle Fluidized Beds: An Experimental Study," *J. Heat Transfer*, vol. 108, No. 4, pp. 907-912, 1986.

1.21. Gelperin, N. I., and Einstein, V. G., "Heat Transfer in Fluidized Beds," in *Fluidization*, edited by J. F. Davidson and D. Harrison, Academic Press, London, 1971.

1.22 Grewal, N. S., and Saxena, S. C., "Heat Transfer between a Horizontal Tube and a Gas-Solid Fluidized Bed," *Int. J. Heat Mass Transfer*, vol. 23, pp. 1505-1519, 1980.

1.23 Genetti, W. E., Schmall, R. A., and Grimmet, E. S., "The Effect of Tube Orientation on Heat Transfer with Bare and Finned Tubes in a Fluidized Bed," *Chem. Eng. Prog. Symp. Ser.*, vol. 67, No. 116, pp. 90-96, 1971.

1.24 Jacob, A., and Osberg, G. B., "Effect of Thermal Conductivity on Local Heat Transfer in a Fluidized Bed," *Can. J. Chem. Eng.*, pp. 5-9, June 1957.

1.25 Kobayashi, M., Ramaswami, D., and Brazelton, W. T., "Heat Transfer from an Internal Surface to a Pulsed Bed," *Chem. Eng. Prog. Symp. Ser.*, vol. 66, No. 105, pp. 58-76, 1970.

1.26 McLaren, J.,and Williams, D. C., "Combustion Efficiency, Sulphur Retention and Heat Transfer in Pilot-Plant Fluidized Bed Combustors," *J. Inst. Fuel.*, vol. 42, pp. 303-308, 1969.

1.27 Mickley, H. S., and Fairbanks, D. F., "Mechanism of Heat Transfer to Fluidized Beds," *AIChE J.*, vol. 1, No. 3, pp. 374-384, 1955.

1.28 Zabrodsky, S. S., *Hydrodynamics and Heat Transfer in Fluidized Beds*, M.I.T. Press, Cambridge, 1966.

1.29 Zabrodsky, S. S., Antonishin, N. V., and Parnas, A. L., "On Fluidized Bed to Surface Heat Transfer," *Canadian J. Chem. Eng.*, vol. 54, pp. 52-58, Feb./April, 1976.

1.30 Martin, H., "Heat Transfer Between Gas Fluidized Beds of Solid Particles and the Surfaces of Immersed Heat Exchanger Elements, Part I and Part II," *Chem. Eng. Process.* vol. 18, pp. 157-169 and 199-223, 1984.

1.31 Yamada, Y., Takahashi, S., and Maki, H., "Study of Gas Combustion Fluidized Beds," *ASME/JSME Thermal Engineering Proceedings*, vol. 4, pp. 499-506, 1991.

1.32 Flamant, G. and Manigault, T., "Combined Wall-to-Fluidized Bed Heat Transfer. Bubbles and Emulsion Contribution at High Temperature," *Int. J. Heat Mass Transfer*, vol. 30, No. 9, pp. 1803-1812, 1987.

1.33 Grewal, N. S., Saxena, S. C., "Experimental Studies of Heat Transfer Between Bundle of Horizontal Tubes and a Gas-Solid Fluidized Bed," ASME Paper 80-HT-119, 1980.

1.34 Gelperin, N. I., Einstein, V. G., and Romanova, N. A., *Khim Prom.*, No. 11, p 823. 1964. Quoted in Gelperin and Einstein (1.21).

1.35 Marooka, S., Maruyama, Y., *et al.*, "Heat Transfer Between Bed and Vertically Inserted Tube Wall, and Holdup of Solid Particles in the Dense and Dilute Zones of a Fluidized Bed," *Heat Transfer Japanese Res.*, vol. 8, No. 1, pp. 61-70, 1979.

1.36 Pata, J., Hartman, M., "Operating Region of Fluidized Bed of Lime and Limestone Particles," *Ind. Eng. Chem.*

Process Des. Dev., vol. 19, pp. 98-103, 1980.

1.37 Zakkay, V., and Miller, G., "Heat Exchanger Designs for Coal-Fired Fluidized Beds,". *Proc. 13th Intersociety Conversion Eng. Conf.*, August 1978, vol. 1, pp. 635-641. Published by Soc. Automotive Engineers., Warrendale, Penn., 1978.

1.38 Saxena, S. C., Grewal, N. S., Gabor, J. D., Zabrodsky, S. S., and Galershtein, D. M., "Heat Transfer between a Fluidized Bed and Immersed Tubes," in *Advances in Heat Transfer*, vol. 14, pp. 149-247, Academic Press, 1978.

1.39 Sakrits, B. V., *Heat transfer from Suspended Beds of Granular Materials to Heat Transfer Surfaces*, Kand.Dissertat., Leningrad, 1959. Quoted in Zabrodsky [1.28].

1.40 Shah, M. M., "Generalized Prediction of Maximum Heat Transfer to Single Cylinders and Spheres in Fluidized Beds," *Heat Transfer Engineering*, vol. 4, No. 3-4, pp. 107-122, 1983.

1.41 Dow, W. M. and Jakob, M., "Heat Transfer Between a Vertical Tube and a Fluidized Air-Solid Mixture," *Chemical Eng. Prog.*, vol. 47, No. 12 pp. 637-648, 1951.

1.42 Vreedenberg, H. A., "Heat Transfer Between Fluidized Beds and Vertically Inserted Tubes," *J. Appl. Chem.*, Supplementry Issue No. 1, pp. S26-S33, 1952.

1.43 Frankel, L. I., Kondukov, N. B., "Intensification of Surface to Bed Heat Transfer in a Fluidized Bed at Controlled Gas Distribution," *Proc. Sixth Int. Heat Transfer Conference*, vol. 6, pp. 37-42, Hemisphere, Washington, 1978.

1.44 Chekansky, V. V., Sheindlin, B. S., Galershtein, D. M., and Antonyuk, K. S., *Tr. Spets. Konstr. Byurs Avtomat. Neftepererab. Neftekhim.*, vol. 3, p. 143 1970. Quoted by Bordulya *et al* [1.12] and Grewal and Saxena [1.33].

1.45 Tabatabaie-Farashahi, M., Upadhyay, S. N., and Saxena, S. C., "Fluidized Bed and Freeboard Heat Transfer Studies with Vertical Tube Bundles," *Letters Heat Mass Transfer*, vol. 8, pp. 91-102, 1981.

1.46 Gelperin, N. I., Einstein, V. G. and Zaikovski, A. V., *Khim Mashinostr.* (Moscow), No. 3, p. 17, 1968. Quoted by Saxena *et al.* [1.38].

1.47 Gelperin, N. I., Einstein, V. G., and Korotyanskaya, A. V., *Int. Chem. Eng.*, vol. 9, p. 137, 1969. Quoted by Saxena *et al* [1.38].

1.48 Figiola, R. S., Suarez, E. G., and Pitts, D. R., "Mixed Particle Size Distribution Effects on Heat Transfer in a Fluidized Bed," *J. Heat Transfer*, vol. 108, pp. 913-915, 1986.

1.49 Mathur, A., and Saxena, S. C., "Correlation of Heat Transfer Between Immersed Surfaces and Gas-Fluidized Beds of Large Particles," *Energy* (Oxford), vol. 11, No. 9, pp. 843-852, 1986.

1.50 Ticshenko, A. T., and Khvastukin, Yu. I., "Fluidized Bed Furnances and Heat Exchangers," *Naukova Dumka*, Kiev, 1973. Quoted by Grewal and Saxena [1.22].

1.51 Shah, P. J., Upadhyay, S. N., and Saxena, S. C., *Heat Transfer from Large Diameter Smooth Horizontal Tubes Immersed in Gas Fluidized Beds*, Paper Presented at the 20th National Heat Transfer Conference, Milwauke, Wisconsin, August 2-8, 1981.

1.52 Saxena, S. C., and Gabor, J. D., "Mechanism of Heat Transfer Between a Surface and a Gas-Fluidized Bed for Combustor Application," *Prog. Energy Combust. Science*, vol. 7, pp. 73-102, 1981.

1.53 Szekeley J., Fisher, R. J., "Bed-to-Wall Radiation Heat Transfer in the Gas-Solid Fluidized Bed," *Chem. Eng. Science*, vol. 24, p. 833, 1968.

1.54 Yoshida, K., Ueno, T., and Kunii, D., "Mechanism of Bed-Wall Heat Transfer in a Fluidized Bed at High Temperatures," *Chem. Eng. Science*, vol. 29, p. 77, 1974.

1.55 Vedamurthy, V. N. and Shastri, M. K., "An Analysis of Conductive and Radiative Heat Transfer to the Walls of Fluidized Bed Combusters," *Int. J. Mass Transfer*, vol. 17, p. 1, 1974.

1.56 Kharachenko, N. V., and Makhorin, K. E., "The Problem of Intensifying Heat Transfer between Fluidized Beds and Submerged Bodies at High Temperatures," Inzh. Fiz. Zhurn, vol. 7, No. 5, pp. 11-17, 1964. Quoted in Zabrodsky [1.28].

1.57 Kolar, A. K., Grewal, N. S., and Saxena, S. C., "Investigation of Radiative Contribution in a High Temperature Fluidized Bed using the Alternate-Slab Method," *International Journal Heat Mass Transfer*, vol. 22, pp. 1695-1703, 1979.

1.58 Botterill, J. S. M., and Sealey, J. C., "Radiative Heat Transfer Between a Gas Fluidized Bed and an Exchanger Surface," *Brit. Chem. Eng.*, vol. 15, p. 1167, 1970.

1.59 Renzhang, Q, Wendi, H., *et al*, "Experimental Research of Radiative Heat Transfer in Fluidized Beds," *Int. J. Heat Mass Transfer*, vol. 30, No. 5, pp. 827-831, 1987.

1.60 Schlapkova, P. Ya., "Heat Transfer to Cylindrical Surface Immersed in a Fluidized Bed at Low Pressure," *J. Eng. Physics*, vol. 10, no. 3, pp. 187-188, 1966.

1.61 Kumada, M., Kume, S., *et al*, "Heat Transport Mechanism Around Single Row Tubes Immersed in Floating Particles," *ASME/JSME Thermal Engng. Proc.*, vol. 4, pp. 529-536, ASME, 1991.

1.62 Wunder, R., "Warmeubergang an vertikalen Warmetauscher- flachen in Gas Wirbelschichten," Dissertation, TU Munich, Germany, 1980. Quoted in Martin [1.30].

1.63 Botterill, J. S., and Denloye, O. O., "Gas Convective Heat Transfer to Packed and Fluidized Beds," *AIChE Sym. Ser.*, vol. 74, no. 176, pp. 194-202, 1978.

1.64 Saxena, S.C., and Joshi, R. K., "Thermal Accomodation and Adsorption Coefficients of Gases," McGraw-Hill, New York, 1989.

I. INTRODUCTION

Fluidized beds are widely used in many industries including chemical processing, metallurgy, and power generation. A fluidized bed consists of solid particles suspended in the fluidizing gas, which is moving vertically upwards through the bed. Heat is added to or removed from the bed either through the walls of the enclosing vessel or through heat exchangers suspended in the bed. Fuel supply and burners are also provided, if needed for the process. Figures 1-1 and 1-2 are schematic representations of fluidized beds with horizontal tube and vertical tube heat exchangers, respectively. Heat transfer to a heat exchanger in a fluidized bed is several times greater than in gas alone or in an unfluidized bed of particles (fixed bed). Fluidized beds also provide good mixing and high mass transfer coefficients.

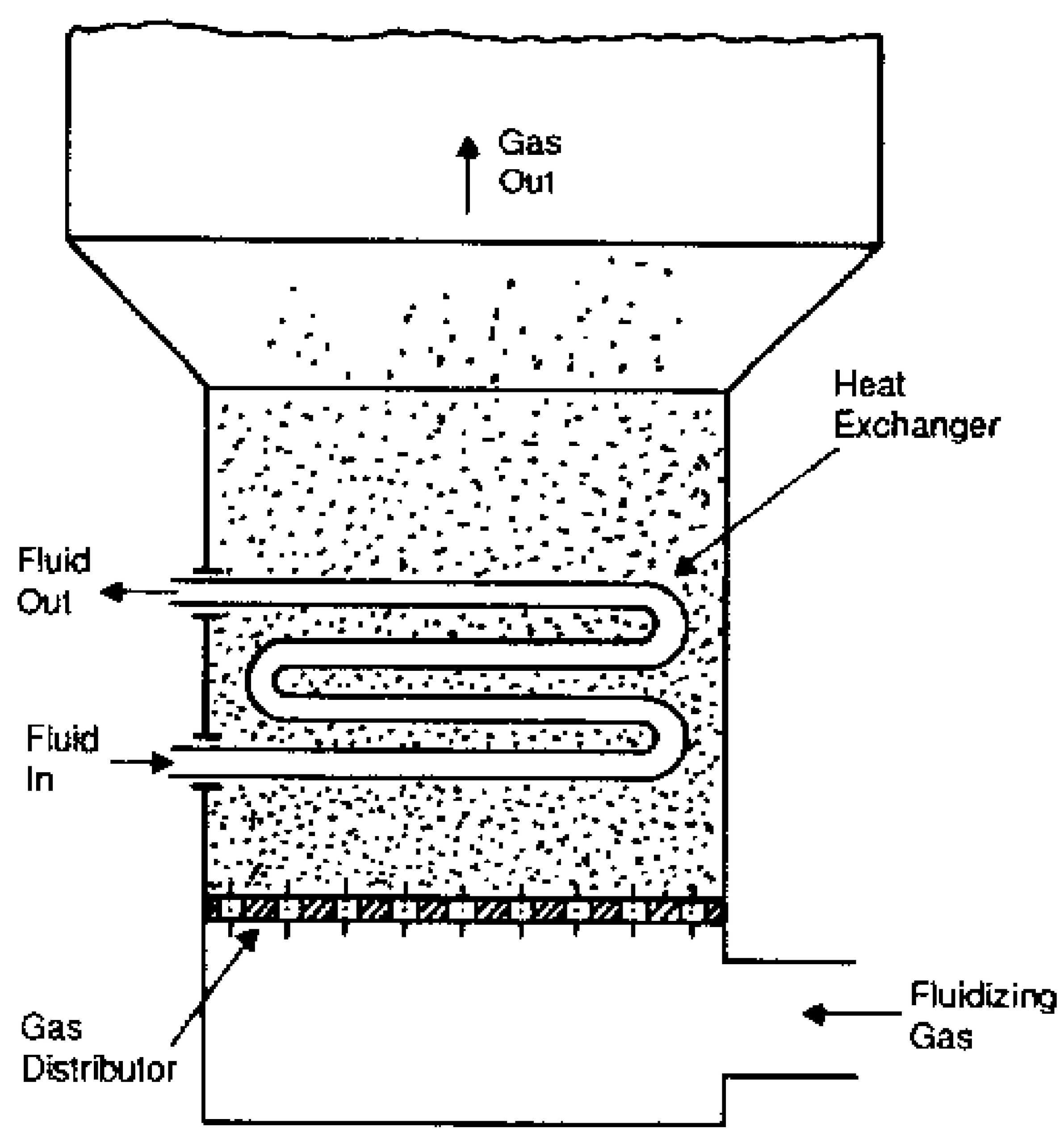

Figure 1-1. Fluidized Bed with Horizontal Tube Heat Exchanger. Fuel and burners are also provided where needed.

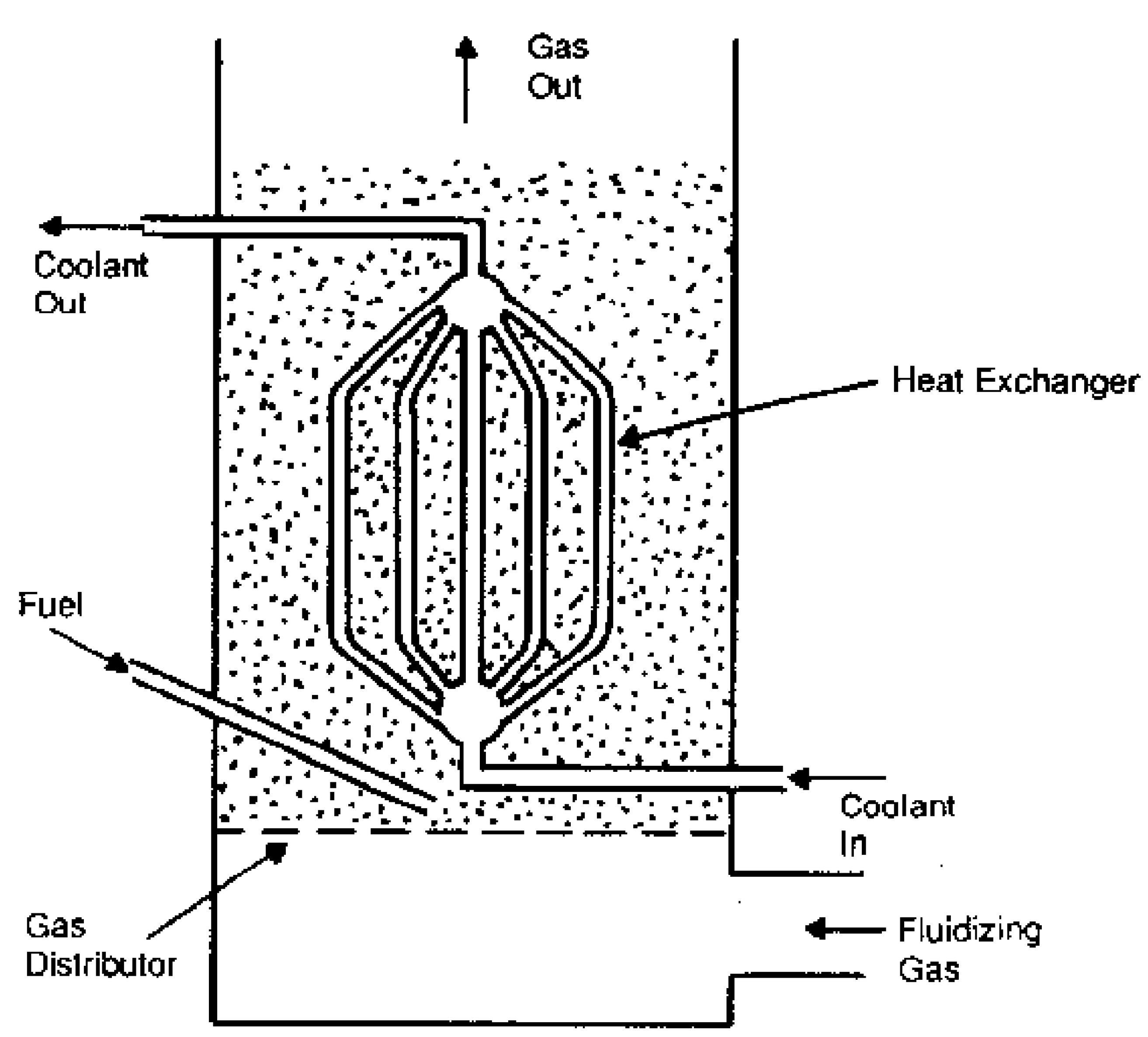

Figure 1-2. Fluidized Bed with Vertical Tube Heat Exchanger and Fuel Supply for Burners

This section deals with heat transfer to plain tubes, bundles of plain tubes, and spheres immersed in gas-fluidized beds. The regime of fluidization considered here is between minimum fluidization and terminal velocities; recirculating beds are not covered.

It should be understood that the heat transfer coefficient at a point on the surface is generally not constant but fluctuates in a cyclic manner. Heat transfer coefficients referred to here are time-averaged values, unless otherwise noted.

Heat transfer coefficients around the circumference of a horizontal tube usually vary. Similarly heat transfer coefficients along the length of a vertical tube and at different points of a sphere often vary. The heat transfer coefficients referred to in further discussions are averaged values. Thus for a horizontal tube, the heat transfer coefficient is:

$$h_w = \frac{1}{2\pi}\int_0^{2\pi} \frac{q}{(T_w - T_b)}\, d\theta \qquad \text{Eq. (1-1)}$$

where θ is the angular position of a point on the tube. For vertical tubes, the integration is carried out over the length, while for a sphere, it is over the total surface area.

Hydrodynamic aspects of fluidized beds are covered in Section 407. Knowledge of some of the topics covered there is essential for understanding the material presented here. Hence its study is advisable.

II. PARAMETERS AFFECTING HEAT TRANSFER

Many parameters have been found to affect heat transfer. Among these are gas velocity, particle size and shape, thermophysical properties of solid and gas, temperature, pressure, tube diameter, height of surface above distributor, bed depth, etc. These are briefly discussed in the following.

Superficial Gas Velocity: Superficial velocity is the velocity based on the total cross-sectional area of the bed. Its typical effect on heat transfer is shown in Figure 1-3. As the velocity increases beyond the minimum fluidization velocity, heat transfer coefficient increases rapidly, reaches a peak and then decreases slowly. The peak heat transfer coefficient is known as the maximum heat transfer coefficient, and the corresponding superficial gas velocity is known as the optimum velocity. At the terminal velocity, particles are blown out and subsequently only gas flows over the heat transfer surface. (For discussions on terminal velocity, see Section 407).

Increasing velocity causes increased mixing and turbulence and thus tends to increase heat transfer. On the other hand, increasing velocity also increases void fraction (also known as bed voidage) and thus decreases the bed density. (For information on void fraction see Section 407). This tends to decrease heat transfer. On the rising branch of the heat transfer curve, the first effect dominates. On the falling branch of the curve, the second effect dominates.

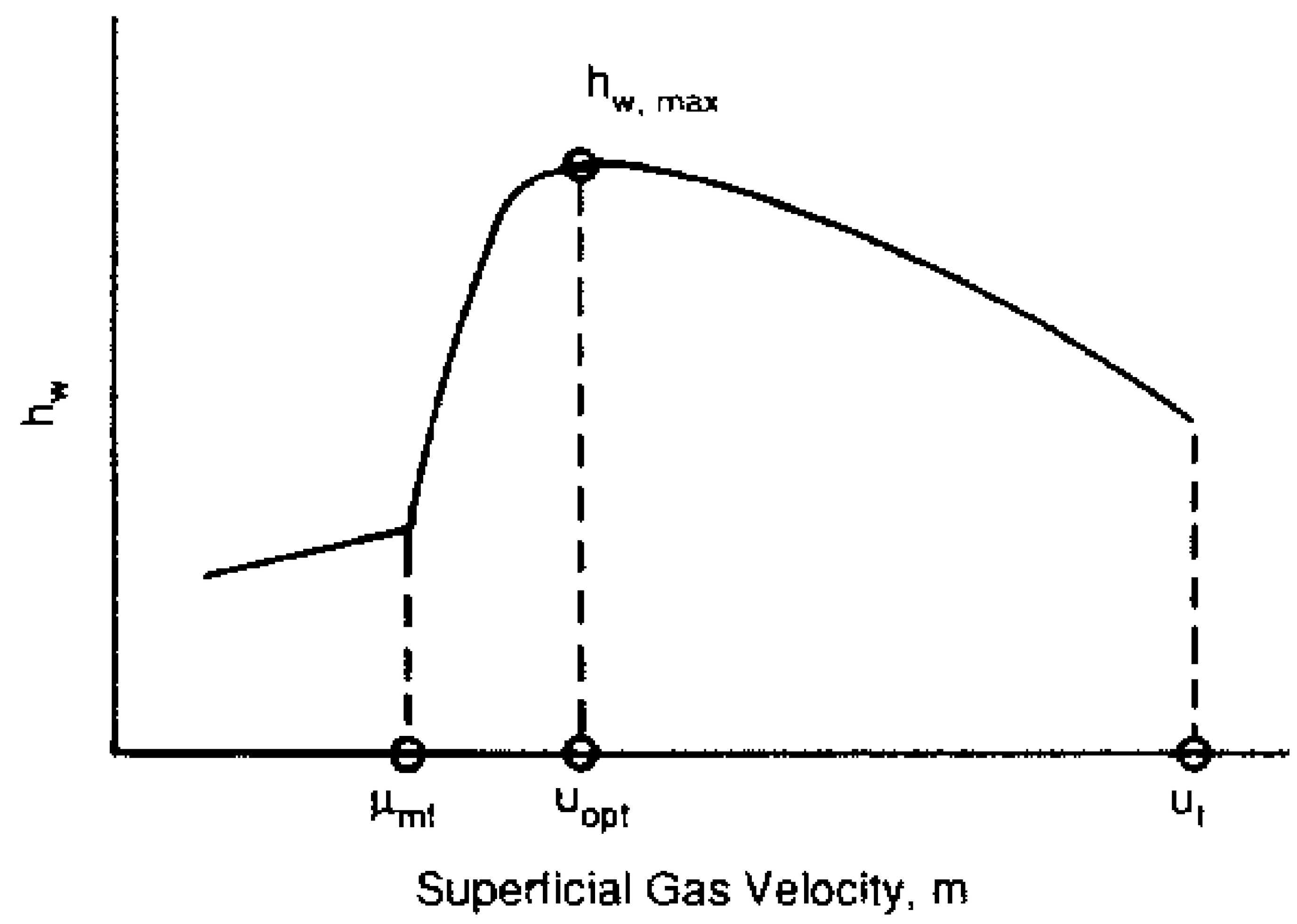

Figure 1-3. Typical Effect of Gas Velocity on Heat Transfer

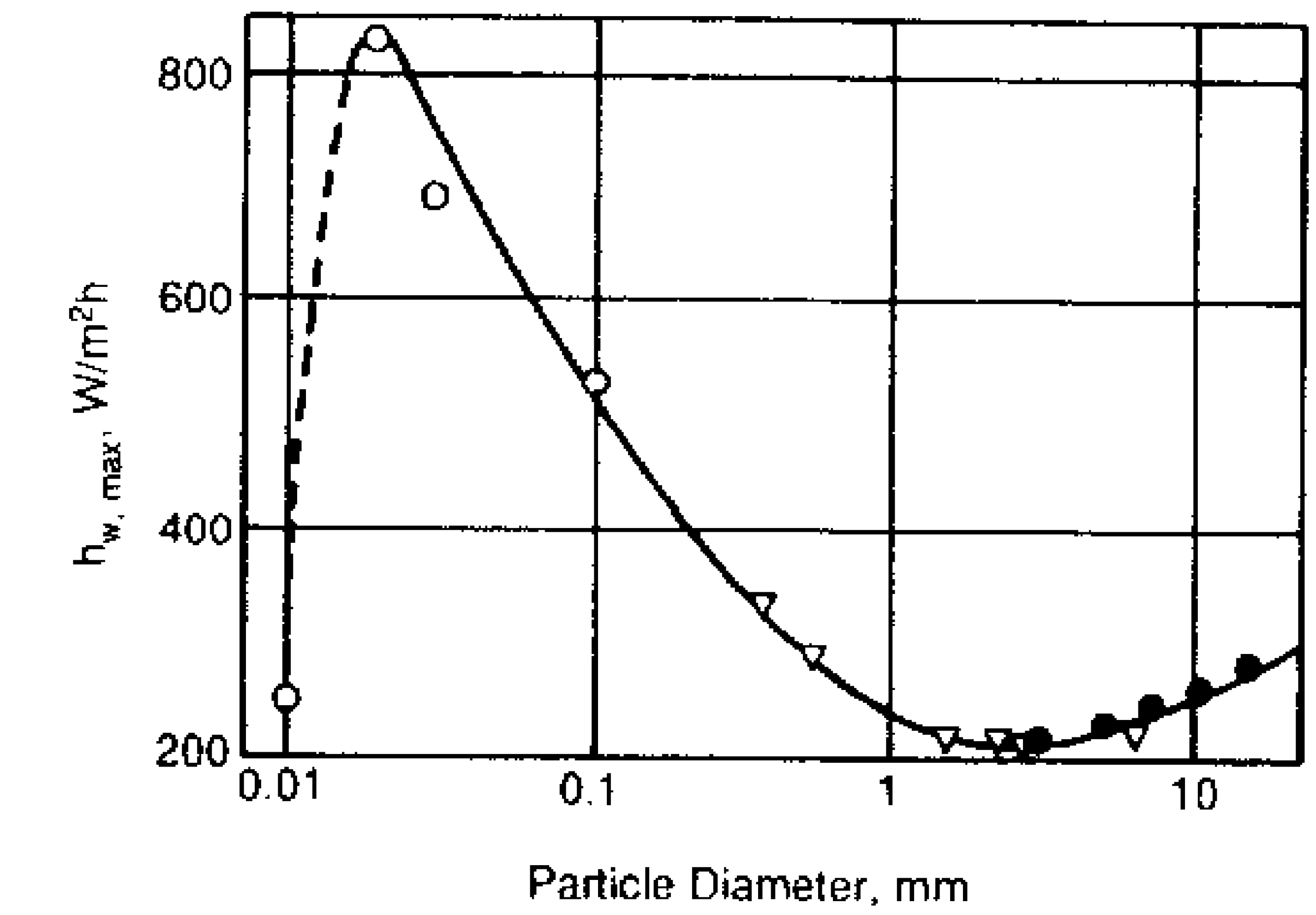

Figure 1-5. Effect of Particle Size on Maximum Heat Transfer to Surface Immersed in Beds of Corundum. Data of Baskakov *et al* [1.9]

Figure 1-4 shows the effect of velocity on particles of various sizes.

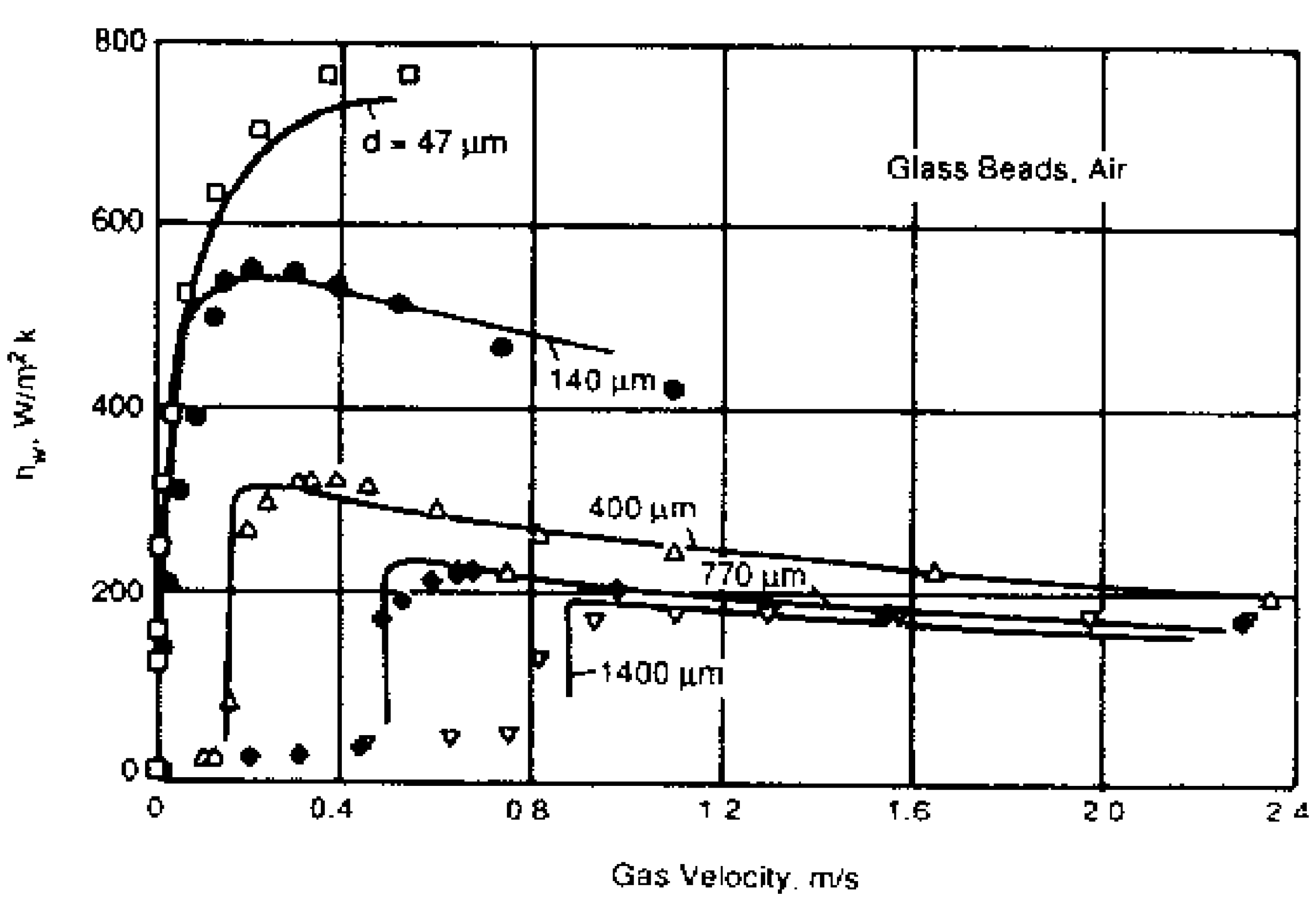

Figure 1-4. Effect of Superficial Gas Velocity on Heat Transfer to Particles of Various Diameters. Data of Wunder [1.62]. The curves are the prediction of Martin Analysis. From Martin [1.30]

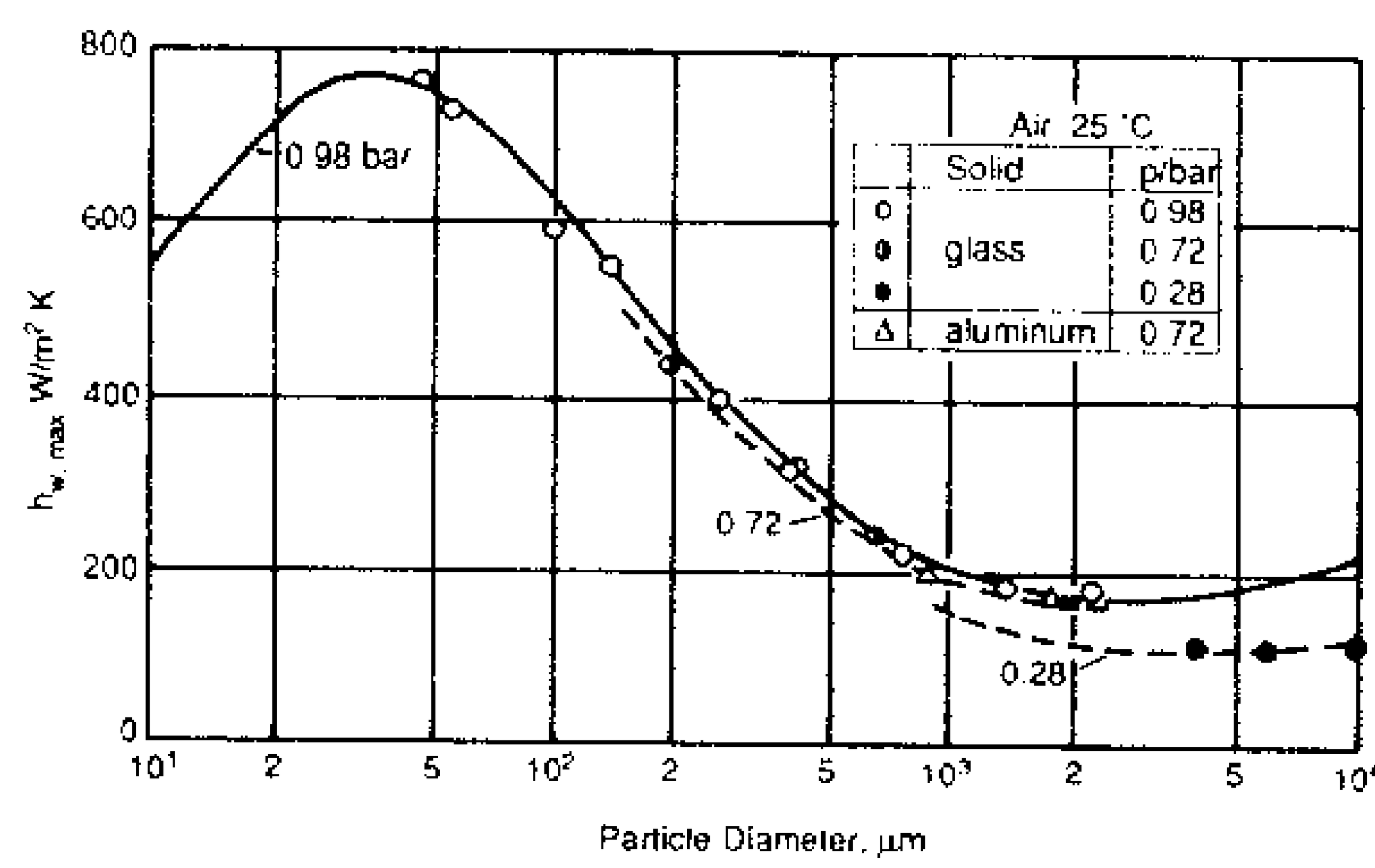

Figure 1-6. Effect of Particle Size and Pressure on Maximum Heat Transfer. Data of Wunder [1.62]. The curves are the prediction os Martin's analysis. From Martin [1.30]

Particle Size: Figures 1-5 and 1-6 show the effect of particle size on maximum heat transfer coefficients. Figure 1-5 is from the tests of Baskakov *et al* [1.9] at atmospheric pressure. Figure 1-6 shows the data of Wunder [1.62] for maximum heat transfer at various pressures. It is noted that for particles of medium size, heat transfer increases with decreasing particle size. Beyond a certain limit, however, heat transfer begins to decrease with decreasing particle diameter. With particles of large size, the heat transfer coefficient increases with increasing particle size. Figure 1-4 shows the effect of particle size over a range of gas velocities.

The effect of particle size on heat transfer can be explained in terms of the penetration theory (see Section III. of this update). As particle size increases, contact area of the particle with the wall decreases while the wall area exposed to gas convection increases. As conduction dominates for small particles, heat transfer initially decreases with increasing particle size. Beyond a certain particle size, virtually all heat transfer is by convection. Then heat transfer increases with increasing particle size because larger particles require higher velocity for fluidization.

Very fine particles tend to agglomerate (form lumps) and therefore have poor fluidization characteristics as indicated by the Geldart classification (see Section 407). This can cause reduction in heat transfer with very fine particles. Addition of coarse particles improves fluidization and heat transfer; see for example Baerns [1.18].

Addition of a small amount of fine particles to medium sized particles can increase heat transfer; see for example Pitts *et al* [1.19].

Studies on mixtures of particles of two sizes have been reported by Figiola *et al* [1.48] and others. Heat transfer tends to be higher than for single-size particles with the mixture-mean diameter.

Cylinder Diameter: Gelperin and Einstein [1.21] concluded that heat transfer coefficient decreases with increasing cylinder diameter when $D_t < 10$ mm but at larger cylinder diameters, D_t has no effect. However, there are many experimental studies contradicting this statement. Shah [1.40] examined a wide variety of data for maximum heat transfer coefficient for surfaces ranging from 0.13 mm wire to 125 mm diameter cylinder. His well-verified correlation shows a continuous decrease in heat transfer coefficient with increasing diameter over the entire range. Figure 1-7 presents some experimental data for maximum heat transfer which show the effect of cylinder diameter.

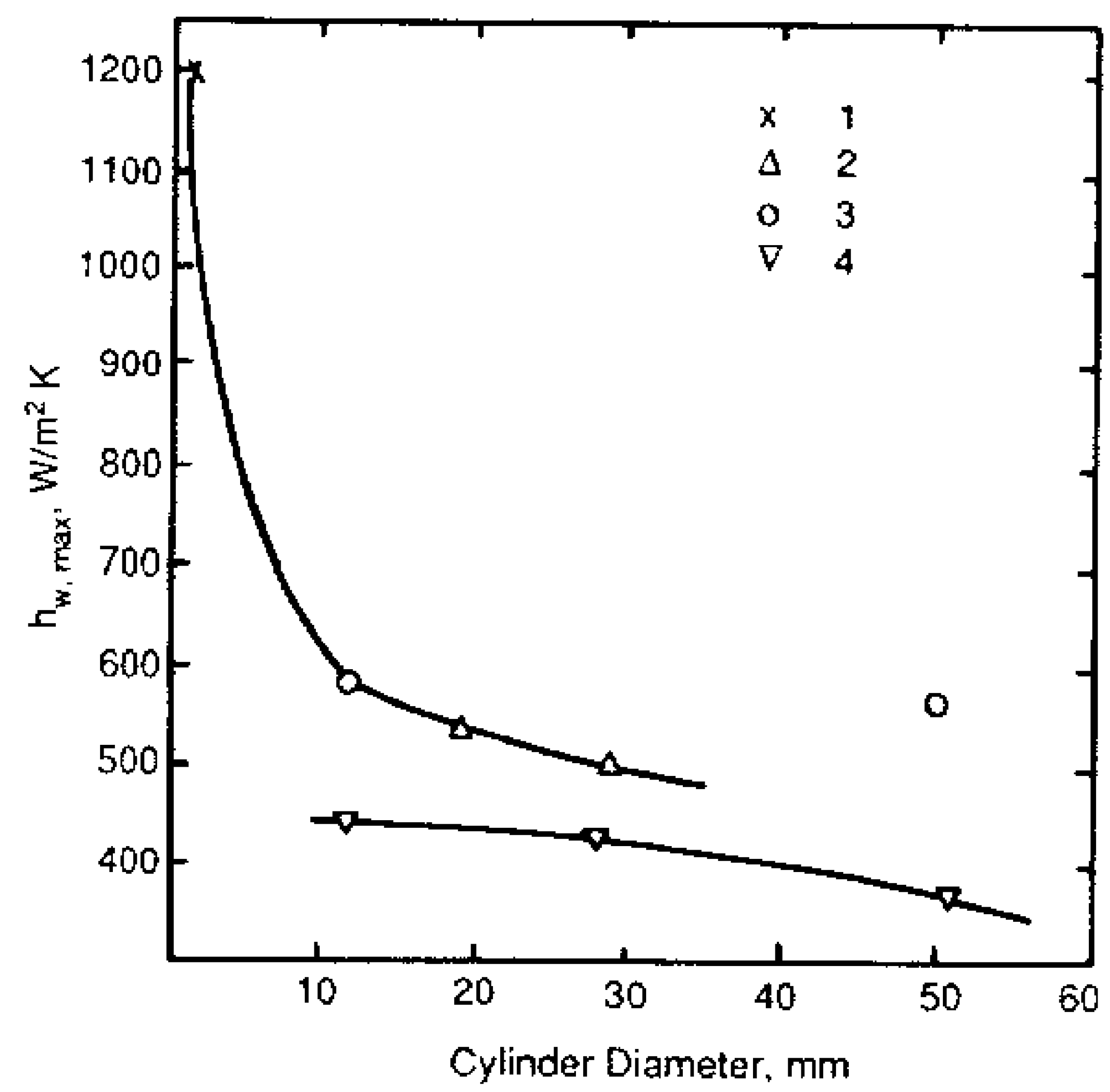

Figure 1-7. Effect of Cylinder Diameter on Maximum Heat Transfer to Glass Beads. 1) Jacob and Osberg [1.24], $D_t = 0.13$ mm, $D_s = 0.292$ mm; 2) Grewal and Saxena [1.22], $D_s = 0.265$ mm; 3) Shah *et al* [1.51], $D_s = 0.265$ mm; 4) Shah *et al* [1.51], $D_s = 0.427$ mm

Turton *et al* [1.16] conducted tests with heated wires of 0.05 to 0.81 mm diameter in beds of particles 0.105 to 0.754 mm. For large wire/small particle beds, h was more than an order of magnitude higher than with air alone. For fine wire/large particle bed, h approached that for wire alone in air.

Pressure: Heat transfer increases as pressure rises above the atmospheric as is shown by many studies such as that of Botterill and Denloye [1.63]. As pressure falls below atmospheric, heat transfer falls as seen in Figure 1-6. In deep vacuum, heat transfer decreases sharply, as reported by Schlapkova [1.60]. The minimum pressure in her tests was 266 N/m^2.

Bed Temperature: Heat transfer increases with increasing temperatures. At moderate temperatures, the increase can be explained as due to convective effects, mainly the increase in gas thermal conductivity. At higher temperatures, much of the increase is attributable to radiation between the bed and the heat transfer surface. This is discussed in more detail in Section II of this update.

Properties of Gas and Solid: Properties of gas that affect single-phase convective heat transfer also affect fluidized bed heat transfer. Among these, thermal conductivity has been found to be the most important, heat transfer increasing with increasing thermal conductivity (e.g. Jacob and Osberg [1.24]).

Among the solid properties, heat transfer increases with increasing specific heat while the effect of thermal conductivity is significant only when k_g/k_s is of the order of 1 or larger [1.30].

Particle Shape and Roughness: In their pioneer experiments, Baerg *et al* [1.10] found that heat transfer coefficients with spherical glass beads were higher than with non-spherical particles. They attributed this to the greater mobility of spherical particles due to their smooth surfaces. Shah [1.40] examined maximum heat transfer data from many sources and concluded that heat transfer coefficients with spherical particles are, on the average, 24 percent higher than with non-spherical particles. Beds of spherical particles have lower voidage than beds of non-spherical particles. The resulting higher bed density may be a factor in higher heat transfer with spherical particles.

Gas Distribution: Frankel *et al* [1.43] carried out a detailed study of gas distribution on heat transfer. Flat velocity profile at bed inlet was found to give much higher heat transfer than convex and jet-like profiles. (Figure 1-8 illustrate types of velocity profiles.) Hence proper gas distributor design is very important.

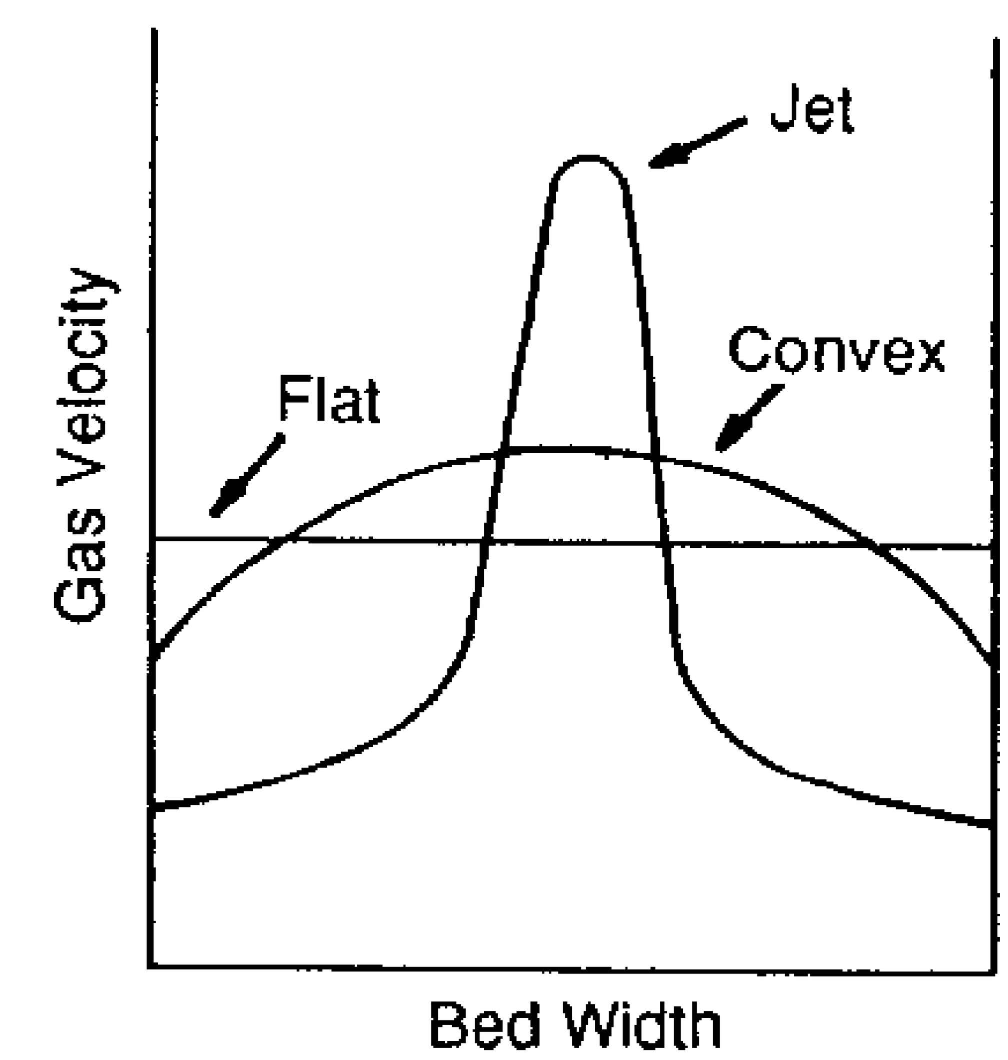

Figure 1-8. Various Profiles of Fluidizing Gas Velocity

Length of Vertical Tube: Many studies have been done on the effect of length of vertical tubes. The results of various studies are apparently conflicting, some indicating a decrease in mean heat transfer with increasing length while some show no effect. The present author's conclusion is that length has an effect only for very short tubes.

Radial Location of Vertical Tube: Conflicting evidence has been presented about the effect of radial location of a vertical tube in a bed. Some such as Vreedenberg [1.42] report a strong influence, peak heat transfer occurring in a non-axial location. Others such as Marooka *et al* [1.35] found minor or no effect of radial location unless the tube is located rather close to the bed wall. The present author's conclusion is that for r/R between 0 to 0.6, variation of heat transfer coefficient does not exceed ± 15% compared to that at r=0. (R is the bed radius and r is distance of the tube from the bed axis).

Vertical Location of Horizontal Tube: Study of test data from several sources indicates that if the tube is positioned one bed diameter above the air distributor or higher, the vertical location has negligible effect. Close to the distributor, heat transfer increases with increasing height above the distributor. Experimental studies include those by Ainstein [1.2] and Abubakr *et al* [1.4].

Tube Inclination: Gelperin and Einstein [1.21] quote experiments in which tube inclination was varied between 0 and 90 degrees. No significant change in heat transfer was found with changing inclination. Genetti *et al* [1.23] found little difference between 0 and 90 degrees but heat transfer coefficients at intermediate angles were found lower. Comparison of data from independent studies on horizontal and vertical tubes shows that heat transfer coefficients for the two are about the same.

Bed Diameter: The correlation of Ainstein [1.2] shows a small decrease in heat transfer with increasing bed diameter while that of Sakrits [1.39] shows the opposite trend. Both correlations are based on their own experimental data. Most other correlations do not include the bed diameter. The conclusion is that the bed diameter's influence on heat transfer is negligible as long as as it is much larger than the immersed heat exchanger.

Bed Height: Experiments by Kobayashi *et al* [1.25] indicate that bed height has no effect. Most other studies lead to the same conclusion.

Miscellaneous Factors: Grewal and Sexena [1.22] found that heat transfer increases slightly with increasing heat flux. This was probably due to the increase in thermal conductivity of air near the tube due to its increased temperature. These authors conducted tests with bronze and copper tubes. No effect of tube material was found.

III. THEORIES OF HEAT TRANSFER

Many theories have been proposed to explain and predict heat transfer between gas fluidized beds and surfaces. For detailed discussions of these theories see Grace [1.1], Botterill [1.17], Xavier and Davidson [1.6], Grewal [1.7], Gelperin and Einstein [1.21], Zabrodsky [1.28] and Saxena *et al* [1.38]. Most of the theories can be put in one of two broad categories, namely the film theory and the penetration theory.

A. FILM THEORY

In the film theory, first proposed by Dow and Jakob [1.41], heat transfer occurs purely by convection between the gas and the heating surface. The role of solid particles is only to enhance the convective heat transfer by thinning the gas boundary layer. The thermophysical properties of solid particles have no effect.

B. PENETRATION THEORY

In the penetration theory, heat transfer occurs both by transient conduction to solid particles and by convection between the gas and the heating surface. Thus thermophysical properties of particles affect heat transfer. In some analyses, particles actually touch the surface while in some, a gap is assumed between the particles and surface. For small particles and low velocities the conduction mechanism dominates while for large particles and high velocities, the convection mechanism dominates. There are two basic versions of the penetration theory. In one version, known as the "packet theory," conduction to packets containing a number of particles is analyzed. Individual particles are not analyzed. In the other version, conduction to "distinct particles" is analyzed. Mechanisms of conduction, convection, and radiation are considered to be independent and additive. Thus:

$$h_w = h_{cond} + h_{conv} + h_{rad} \qquad \text{Eq. (1-2)}$$

$$q = h_w (T_b - T_w) \qquad \text{Eq. (1-3)}$$

In the film theory, $h_{cond} = 0$.

C. PACKET THEORY

The packet theory was originated by Mickley and Fairbanks [1.27]. In their model, heat transfer occurs by packets of particles that contact the heat transfer surface, exchange heat with it and then move back into the bed core to give up the heat gained at the surface. The packets have the properties of the quiescent bed (voidage, thermal conductivity, specific heat). The packets are not permanent but have a finite life. Calculations with this model involve several factors that are difficult to estimate. The packet model has been further developed by other researchers such as Baskakov *et al* [9] and Kumada *et al* [1.61] but still the equations involve several factors that have to be adjusted to fit the heat transfer data from various sources.

D. SINGLE PARTICLE AND CHAIN OF PARTICLE MODELS

While the packet theory considers conduction to a collection of particles, this version of the penetration theory considers heat conduction to individual solid particles. Some of the analyses consider only one particle in contact with the heating surface (or close to it) and neglect heat transfer from this particle to other particles in contact with it. When chains of particles are considered, unsteady heat conduction occurs from the surface to the first particle of the chain nearest to it. The heat is conducted from the first particle to the next particle and so on until the particle temperature approaches the bed temperature. Analyses have been presented using chains of 2 particles, 4 particles, and an infinite number of particles. Gaps of various dimensions are assumed between the surface and the particles, and in between the particles of the chain. Other adjustable factors are also often involved.

IV. PREDICTIVE TECHNIQUES FOR SINGLE CYLINDERS AND TUBES

A. ANALYTICAL METHODS

Numerous solutions have been proposed based on the theories mentioned in Section 3. The analytical solution by Martin (following) has had extensive verification

Martin's Analytical Solution - Martin's model [1.30] considers transient conduction to single particles that exchange heat with the cooling (or heating) surface for a short time by conduction through a thin air gap then move back into the bulk of the bed. This air gap is a modified mean free path of gas molecules calculated using the kinetic gas theory. (According to Martin, the gap in the vicinity of the contact point between particle and heater surface is always less than the mean free path.). The following relations were developed by him for the conduction component of heat transfer (called "particle convection" by him):

$$\frac{h_{cond}\, D_s}{k_g} = (1-\varepsilon)\, Z\left(1-e^{-N}\right) \qquad \text{Eq. (1-4)}$$

$$Z = \frac{1}{6}\frac{\rho_s C_s}{k_g}\left[\frac{g D_s^3 (\varepsilon-\varepsilon_{mf})}{5(1-\varepsilon_{mf})(1-\varepsilon)}\right]^{0.5} \qquad \text{Eq. (1-5)}$$

$$N = Nu_{ws}/(BZ) \qquad \text{Eq. (1-6)}$$

$$\frac{1}{Nu_{ws}} = \frac{1}{NU_{ws,max}} + \frac{k_g/k_s}{4\left[1+\left(\dfrac{3Bk_gZ}{2\pi k_s}\right)^{0.5}\right]} \qquad \text{Eq. (1-7)}$$

The left hand side of Eq. (1-7) represents the overall resistance to heat transfer between a particle and the heater surface during their contact; the first term on the right hand side represents the resistance through the air gap while the second term on the right hand side represents the internal transient conduction resistance of the particle.

$$Nu_{ws,max} = 4\left[\left(1+\frac{2\xi}{D_s}\right)\ln\left(1+\frac{D_s}{2\xi}\right)-1\right] \qquad \text{Eq. (1-8)}$$

$$\xi = 2\left(\frac{2}{\gamma}-1\right)\frac{k_g(2\pi RT/M)^{0.5}}{p\left(2C_{pg}-R/M\right)} \qquad \text{Eq. (1-9)}$$

ξ is the modified mean free path and γ is the accommodation coefficient whose values at 25 °C are as follows:

Gas	γ
H_2	0.2
He	0.235
Ne	0.573
H_20	0.80
Ar	0.876
air, CO_2	0.90
Kr	0.933
Xe	0.956
NH_3	0.90
CH_4	0.70

At other temperatures, γ can be calculated by:

$$\log\left(\frac{1}{\gamma}-1\right) = 0.6 - \frac{1000/T_g+1}{C_A} \qquad \text{Eq. (1-10)}$$

The value of C_A is readily calculated using the known value of γ at 25 °C. For additional information on accommodation coefficients, see Saxena and Joshi [1.64].

The convective heat transfer coefficient is calculated by the following empirical correlation of Baskakov *et al* [1.9]:

$$h_{conv} D_s/k_g = 0.009\, Pr^{1/3}\, Ar^{1/2}\, (u/u_{opt})^n \qquad \text{Eq. (1-11)}$$

where $n = 0.3$ for $u < u_{opt}$ and $n = 0$ for $u > u_{opt}$

The only unknown in the above set of equations is the factor B. This was calculated to be 2.6 by analyzing test data for glass beads from one source, and the same value was recommended for general use.

The radiation heat transfer coefficient was calculated as explained in Section 6.

The range of parameters covered in the analysis of Martin is listed in Table 1-1. It is seen that the range is very wide except that the tube diameter range is restricted to 6.35-40 mm. As experimental data clearly demonstrate an influence of diameter while it is not included in the equations of Martin's model, caution should be exercised in using it outside the verified range of diameter. For methods to calculate bed voidage, see Section 407.

Figures 1-4 and 1-6 compare the predictions of this analysis with some experimental data. Good agreement is seen.

Table 1-1
Verified Range of Martin's Analytical Solution

Tube dia., mm	6.35 - 40
Tube orientation	Horizontal, Vertical
Sphere dia., mm	9.5 - 60
Particle dia., mm	0.004 - 10.0
ρ_s, kg/m^3	26 - 11,180
p, bar	0.3 - 25
k_g, W/m K	0.035 -240
k_s, W/m K	25 - 960

B. EMPIRICAL CORRELATIONS

Discussions here are confined to moderate temperatures at which the contribution of radiation is negligible. Calculation of radiation contribution is discussed in Section 6.

Numerous correlations have been proposed for predicting the maximum heat transfer coefficient as well as heat transfer over a range of velocities. Most of these have a very narrow range of applicability. A few that have been shown to agree with a wide range of data are discussed here. Many other correlations are listed in Grewal [1.7] and Saxena *et al* [1.38].

The following dimensional equation of Zabrodsky [1.28] for maximum heat transfer has found wide acceptance:

$$h_{w,max} = 35.7\, k_g^{0.6}\, D_s^{-0.36}\, \rho_s^{0.2} \qquad \text{Eq. (1-12)}$$

The units are as listed in the "Symbols" section. While this correlation has been found to agree with a wide variety of data, some of its limitations are apparent from the discussions in Section 2 as a number of parameters known to affect heat transfer are absent from this formula. Perhaps the most notable point is that it predicts a continuous decrease in heat transfer with increasing particle size, while the actual trend is different, as seen in Figure 1-5. The measurements of Jacob and Osberg [1.24] on a 0.13 mm wire are about four times higher than the Zabrodsky correlation. Martin [1.30] has also reported some cases in which the Zabrodsky formula failed.

Zabrodsky *et al* [1.29] have shown that for particle densities in the range of 2000 to 4000 kg/m^3, and air as the fluidizing medium, Eq. (1-12) can be approximated by the following dimensionless relation:

$$Nu_{s,max} = 0.88\, Ar^{0.213} \qquad \text{Eq. (1-13)}$$

A well-verified correlation for maximum heat transfer is that of Shah [1.40]. It is described by the following equations:

For $Re_{opt} < 170$,

$$Nu_{t,max} = 8.55\, F\, Re_{opt}^{0.158} \left(\frac{D_t}{D_s}\right)^{0.805} \left(\frac{C_s}{C_{pg}}\right)^{0.18} Pr_g^{0.33} \qquad \text{Eq. (1-14)}$$

For $Re_{opt} > 170$,

$$Nu_{t,max} = 0.52\, F\, Re_{opt}^{0.695} \left(\frac{D_t}{D_p}\right)^{0.805} Pr_g^{0.33} \qquad \text{Eq. (1-15)}$$

The parameter F = 1.24 for spherical particles and F = 1.00 for non-spherical particles. All properties are calculated at the bulk gas temperature.

This correlation was developed by analyzing data from 33 independent studies covering a very wide range as seen in Table 1-2. The mean deviation for all data was 17%.

Table 1-2
Range of Data with which the Shah Correlation for Maximum Heat Transfer Was Verified
Eqns. (1-14) and (1-15), From Shah [1.40]

Geometry	Single spheres and single horizontal and vertical cylinders
Gases	Air, CO_2, helium, R-12, hydrogen
D_s, µm	104-15,000
D_t,mm	0.13-220
Pressure, bars	1-9.25
Bed temperature, °C	22-900
ρ_s, kg/m^3	1986-11340
$\rho_s C_s$, kJ/m^3 °C	1474-4173
Ar	28-4.5 x 10^8
Re_{opt}	0.04-4800
C_s/C_g	0.053-1.2

Many methods for calculating Re_{opt} are available; see for example Zabrodsky [1.28] and Pata and Hartman [1.36]. Shah [1.40] compared several of them against the data. None of them was entirely satisfactory. However, for $Re_{opt} < 170$, Shah found the following formula of Tode [1.3] to give the best results for horizontal tubes and spheres:

$$Re_{opt} = \frac{Ar}{18 + 5.22\,Ar^{0.5}} \qquad \text{Eq. (1-16)}$$

For $Re_{opt} > 170$, all the data analyzed by Shah were for vertical tubes. None of the available correlations for optimum velocity was found satisfactory. He therefore fitted the following relation to all data for vertical tubes (including $Re_{opt} < 170$):

$$Re_{opt} = 0.065\,Ar^{0.58} \qquad \text{Eq. (1-17)}$$

It gave satisfactory agreement with the data for air-fluidized beds but considerably over-predicted helium data at low Reynolds numbers.

The foregoing correlations are for maximum heat transfer. For heat transfer over a range of velocities, Grewal and Saxena [1.22] gave the following correlation:

$$Nu_t = 47(1-\varepsilon)\left(\frac{G\,D_t\,\mu_g}{\rho_s\,\rho_g\,D_s^{\,3}\,g}\right)^{0.325} \left(\frac{\rho_s\,C_s\,D_t^{\,1.5}\,g^{0.5}}{k_g}\right)^{0.23} Pr_g^{\,0.3} \qquad \text{Eq. (1-18)}$$

All properties are taken at the film temperature. This correlation was verified with data for single horizontal tubes in air-fluidized beds from several sources. These included D_s from 0.167 to 0.504 mm, D_t from 12.7 to 28.6 mm, particle density from 2490 to 4450 kg/m^3, and particle specific heat from 0.44 to 0.929 kJ/kg K. However, Shah *et al* [1.51] found it to greatly underpredict their data for a 50.8 mm diameter tube.

A correlation for large particles (Ar > 130,000) has been presented by Mathur and Saxena [1.49]. It shows agreement with data from several sources for particle Reynolds numbers up to 5,000 and D_s from 0.62 to 4.0 mm. They tested several other correlations and found them to give poor agreement with the same data.

C. DESIGN RECOMMENDATIONS

1. The analytical solution of Martin, Eqns. (1-4) through (1-11), is recommended for general use in its verified range, listed in Table 1-1.

2. The Shah correlation for maximum heat transfer, Eqns. (1-14) and (1-15), is recommended in its verified range, listed in Table 1-2.

3. For tube diameters beyond the verified range of Martin's analysis, it is recommended that the values of h_w predicted by Martin's analysis at various velocities be multiplied by the ratio of $h_{w,\,max}$ predicted by the Shah correlation and the Martin analysis.

V. TUBE BUNDLES

The majority of heat exchangers used in practice are bundles of tubes. Hence their study is of great practical importance. Only bundles of plain tubes are discussed here.

The bundle may consist of horizontal, vertical, or inclined tubes. Most of the studies have been done on horizontal or vertical tube bundles. In horizontal bundles, tubes may be arranged inline or staggered. The pitch P is defined as the length of a straight line joining the centers of two adjacent tubes (some authors have defined it differently, but this is the definition used here). A common arrangement of the staggered bundles is that in which tube centers are at the corners of equilateral triangles. Horizontal and vertical pitch (P_H and P_V) are defined as shown in Figure 1-9. In an equilateral triangle arrangement, $P_V = (0.866\,P_H)$ and $P = P_H$.

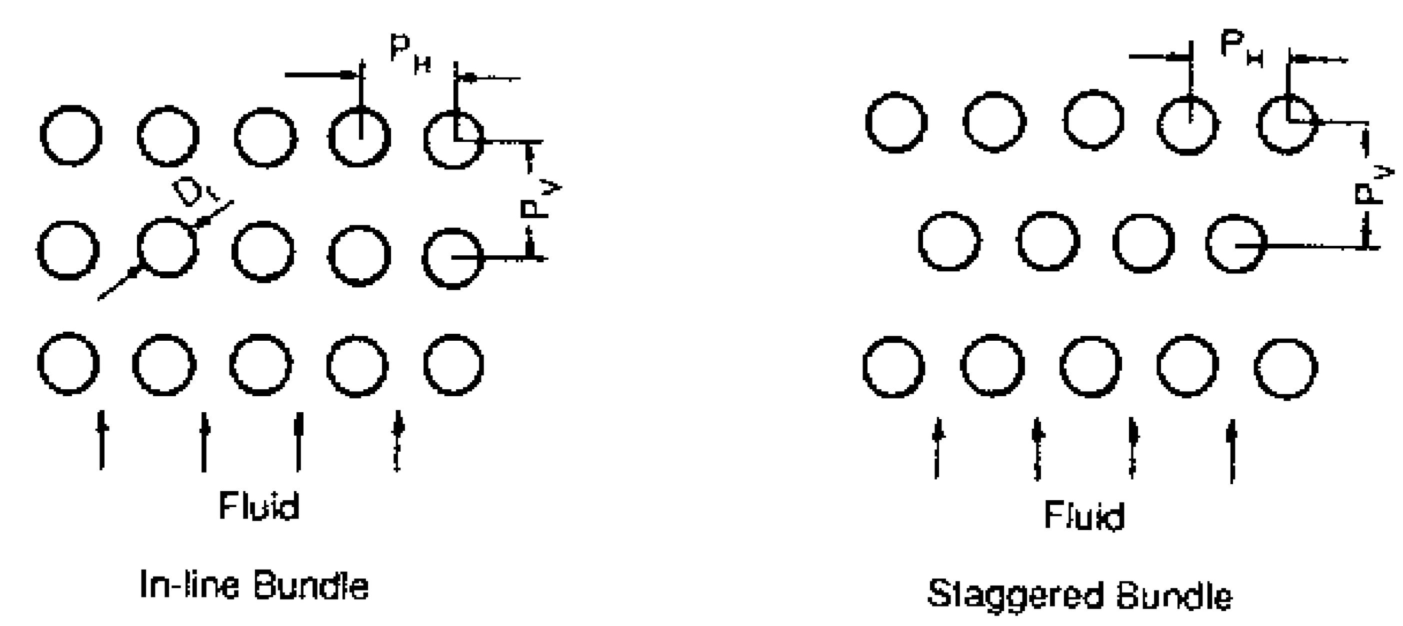

Figure 1-9. Horizontal Tube Bundle Arrangements and Definitions

Various experimental studies and correlations on tube bundles have been reviewed, among others, by Saxena *et al* [1.38] and Gelperin and Einstein [1.21].

A. Horizontal Tubes Bundles

Gelperin *et al* [1.46, 1.47] experimented with staggered and inline bundles of 20 mm diameter tubes. Particles of quartz sand (D_s = 160, 260, and 350 μm) were fluidized by air. For the in-line bundles, P_H/D_t varied from 2 to 9 and P_V/D_t varied from 1 to 8. In the staggered bundle, the relative horizontal pitch was the same but P_V/D_t varied from 0 to 10. Heat transfer in the central tubes was found to be 5 to 7% lower than for the lowest tubes. In the inline bundle, vertical pitch was found to have no effect except when the tubes almost touched. Horizontal pitch was found to have a fairly strong effect. In the staggered bundle both the horizontal and vertical pitch were found to have effect. Their correlations for maximum heat transfer to staggered bundles of five and nine rows may be written as:

Staggered bundles, P_H/D_t = 2 to 9, P_v/D_t = 0 to 10,

$$h_{w,max} / h_{st} = 1.1\left[1-\frac{D_t}{P_H}\left(1+\frac{D_t}{P_v+D_t}\right)\right]^{0.25} \qquad \text{Eq. (1-19)}$$

In-line bundles, P_H/D_t = 2 to 9, P_v/D_t = 1 to 8,

$$h_{w,max} / h_{st} = 1.05\left(1-\frac{D_t}{P_H}\right)^{0.25} \qquad \text{Eq. (1-20)}$$

where h_{st} is the maximum heat transfer coefficient for a single tube.

Grewal and Saxena [1.22] experimented with staggered bundles of tubes of diameter 12.7 and 28.6 mm arranged in equilateral triangular fashion. The range of parameters in their study is listed in Table 1-3. Heat transfer coefficients with three and five rows were found to be the same. Data were correlated by the following relation within ± 13% (P/D_t from 1.75 to 9).

$$h_{w,\,max} = h_{st}\ [1\text{-}0.21\ (P/D_t)\text{-}1.75] \qquad \text{Eq. (1-21)}$$

Bansal *et al* [1.8] correlated their own data for a staggered bundle with P/D_t between 1.1 and 1.3 by the following relation:

$$h_w = h_{st}\ 1\text{-}0.4\ [\ (P/D_t)\text{-}2.5] \qquad \text{Eq. (1-22)}$$

McLaren and Williams [1.26] have suggested that it is the minimum gap between the tubes ($\delta = P\text{-}D_t$) that is the important parameter rather than P/D_t used in most correlations. Their experiments were done with staggered bundles of 35 and 60 mm tubes. Gaps varied from 10 to 300 mm.

The correlation of Chekansky *et al* [1.44] considers the minimum gap between the tubes and is as follows:

$$h_{w,\,max} = 28.2\ \rho_s^{0.2}\ k_g^{0.6}\ D_s^{-0.36}\ (20/D_t)^{0.12}\ (\delta/D_s)^{0.04} \qquad \text{Eq. (1-23)}$$

It is stated to be applicable for δ/D_s = 16 to 63. δ is the narrowest gap between tubes. The dimensions are ρ_s in kg/m^3, D_s in m, D_t in mm, and k_g in W/m °C. It is based on data for staggered bundles with D_t = 50 and 102 mm. Solids were porous corundum D_s = 0.44 mm and quartz sand D_s = 0.22 mm.

Goshayeshi *et al* [1.20] experimented with bundles of tubes arranged in equilateral triangles with P/D_t = 3, D_t = 50.8 mm, and particles of 2.14 and 3.33 mm diameter. Heat transfer coefficients were about the same as on single tubes.

Shah [1.40] compared his correlation for maximum heat transfer to single tubes with data from several tube bundles. The data sets analyzed are listed in Table 1-3. All these data were predicted adequately without any correction factor. On the basis of these data as well as study of data from other sources, Shah concluded that if P/D_t exceeded 3, correlations for single tubes can be confidently applied to tube bundles. For smaller pitch, the possibility of some correction may be considered.

Bordulya *et al* [1.12] experimented with in-line and staggered bundles (equilateral triangular arrangement) made from 14 mm OD tubes. Particles of sand (D_s = 0.25 and 0.66 mm) were fluidized by air. P/D_t varied from 2 to 6. All measurements were within + 20% of Eq. (1-23) and within ± 8.5% of Eq. (1-21). Measurements on staggered bundles exceeded Eq. (1-19) by 25 to 50%. Measurements on in-line bundles exceeded Eq. (1-20) by 12 to 25%.

Table 1-3
Experimental Studies on Heat Transfer to Horizontal Bundles in Air Fluidized Beds. The maximum heat transfer coefficients from these studies agree with the correlation of Shah, Eq. (1-14) and (1-15). From Shah [1.40].

Source	Geometry	D_t mm	P_H/D_t	P_v/D_t	Solid	D_s µm	T_b °C
Grewal & Saxena [1.33]	Staggered	12.7 28.6	1.75 9.0	1.51 7.8	Silica sand, alumina	167 504	room
Zabrodsky *et al* [1.13]	in-line	30.0	2.00 3.33	2.00 3.33	millet, fireclay	2000 3000	room
Bordulya *et al* [1.12]	in-line staggered	14.0	2.0 4.0	1.73 4.00	sand	250 660	room
Ku *et al* [1.14]	staggered	30.8	3.3	1.85	silica sand	1080 1200	315 815
Golan and Cherrington [1.15]	staggered	101.6	0.94	0.81	sulfate dolomite	274 874	274 843

Grewal and Saxena [1.33] compared their data listed in Table 1-3 with two correlations. Eq. (1-19) predicted all data within + 10 and - 20%. Eq. (1-23) predicted all data within + 15 and - 5%.

Figure 1-10 compares some of the correlations for staggered bundles, assuming equilateral triangular arrangement. It is seen that the correlation of Bansal *et al*, Eq. (1-22), agree well with that of Grewal and Saxena, Eq. (1-21), in the latter's verified range. The correlation of Gelperin *et al* predicts lower values. It was noted in the previous paragraph that it generally underpredicts experimental data. Hence Eq. (1-22) seems to be the most reliable.

B. VERTICAL TUBE BUNDLES

Saxena *et al* [1.38] have reviewed the experimental data of several researchers on bundles of vertical tubes. They concluded that heat transfer increases with tube spacing and is greater for tubes located near the center of the bundle, at least for the small units.

Gelperin and Einstein [1.21] have quoted experiments on bundles of vertical tubes with D_t from 20 to 40 mm, arranged in a triangular manner with $P/D_t = 1.25$ to 5. Quartz sand of various sizes was fluidized by air. It was found that when P/D_t decreased from 5 to 2, $h_{w,\,max}$ dropped by 5 to 7%. When P/D_t decreased to 1.25, $h_{w,\,max}$ dropped by 15 to 20%. On the rising branch of h_w - u curve, particularly close to minimum fluidization, close spacing of tubes resulted in a decrease of 35 to 50 %. Their observations on maximum heat transfer are in good agreement with Eq. (1-22). The above, together with the data shown graphically in [1.21], suggests that Eq. (1-22) can be used for u/uopt > 0.75. Based on these data, Gelperin and Einstein [1.21] recommend the following relation for P/D_t = 1.5 to 5.

$$Nu_{s,\,max} = 0.75\,Ar^{0.22}\left[1-\frac{D_t}{P}\right]^{0.14} \qquad \text{Eq. (1-24)}$$

Gelperin *et al* [1.34] recommend the following correlation for vertical tube bundles:

$$Nu_{s,\,max} = 0.64\,Ar^{0.22}\,(P/D_t)^{0.09} \qquad \text{Eq. (1-25)}$$

The data on which this correlation was based included particles of 0.16 to 0.35 mm, Ar from 210 to 2400, and P/D_t from 2 to 5. The tube diameter in these tests appears to have been 20 to 40 mm.

Experiments on vertical tube bundles have been reported by Tabatabaie-Farashahi [1.45] and Zakkay and Miller [1.37], among others, but no correlation or comparison with other predictive techniques is given.

C. DESIGN RECOMMENDATIONS

The following recommendations are made for calculation of average heat transfer to tube bundles immersed in fluidized bed:

1. For horizontal bundles in equilateral triangular arrangement, Eq. (1-22) is recommended for $P/D_t \geq 1.1$. It can be used at any velocity, including u_{opt}.
2. For other staggered bundles of horizontal tubes, Eq. (1-19) may be used to calculate maximum heat transfer coefficients.
3. For in-line horizontal bundles, Eq. (1-20) may be used to calculate maximum heat transfer coefficient.

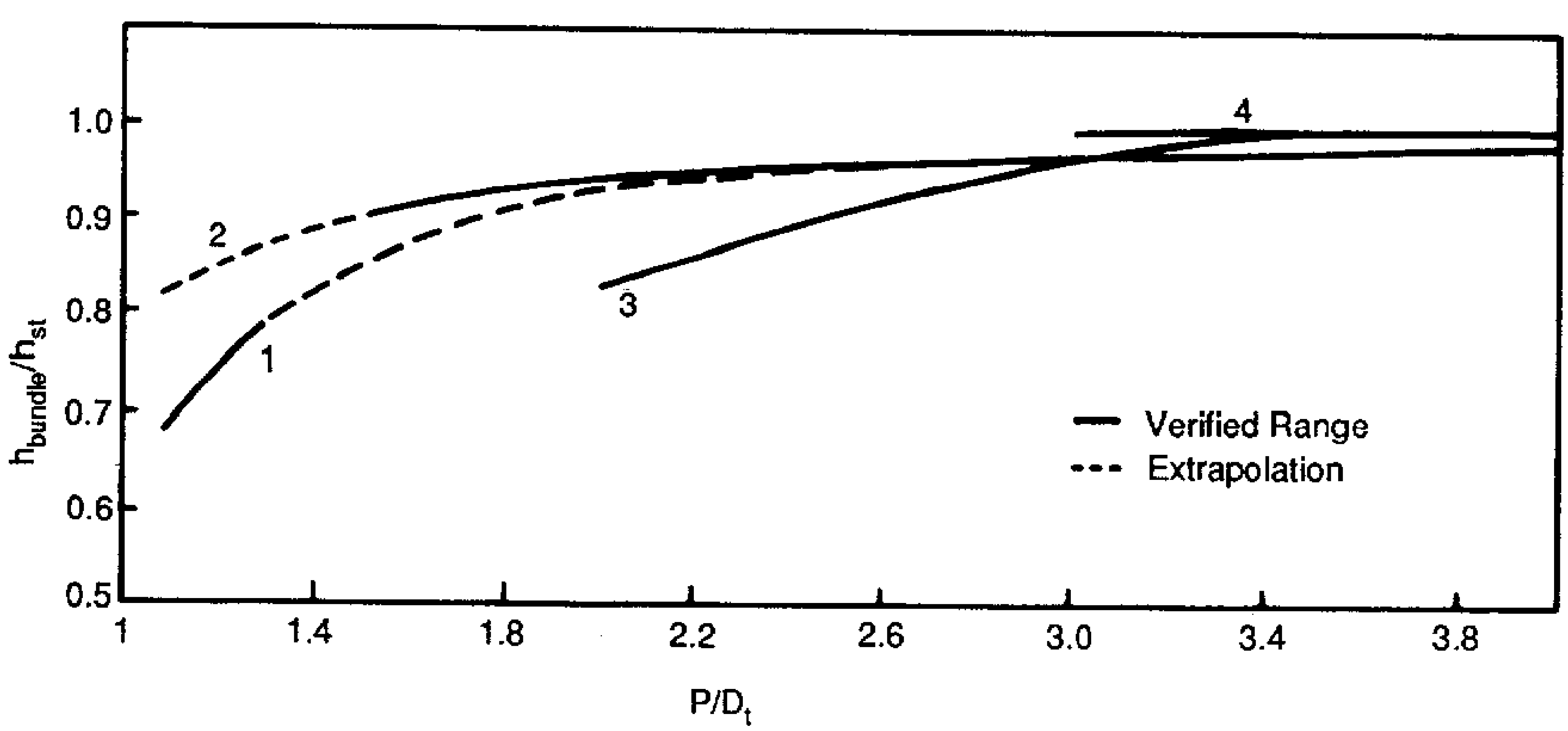

Figure 1-10. Comparison of Various Correlations for Heat Transfer to Horizontal Tube Bundles with Equilateral Triangle Arrangement. 1 - Eq. (1-22), 2 - Eq. (1-21), 3 - Eq. (1-19), 4 - Recommendation by Shah [1.40]

4. For vertical tube bundles, Eq. (1-22) may be used for $P/D_t \geq 1.25$ and $u \geq 0.75\ u_{opt}$. Considerable caution should be exercized as this recommendation is based on limited data.

5. For recommendations on calculation of single tube heat transfer coefficient h_{st}, see Section IV. of this update.

VI. RADIATION HEAT TRANSFER

A. RADIATION HEAT TRANSFER COEFFICIENT AND EFFECTIVE EMISSIVITY

The bed-to-surface radiation heat transfer may be calculated by:

$$q = h_{rad}\,(T_b - T_w) \qquad \text{Eq. (1-26)}$$

h_{rad} is the radiation heat transfer coefficient defined as:

$$h_{rad} = \frac{\sigma\, e_{eff}\left(T_b^4 - T_w^4\right)}{\left(T_b - T_w\right)} \qquad \text{Eq. (1-27)}$$

σ is the Stefan-Boltzman constant and e_{eff} is the effective bed-to-surface emissivity. It includes the effects of the emissivities of bed and surface and view factors. Furthermore, the particles close to the surface are cooler than the bed. Hence e_{eff} also includes this effect unless T_b is replaced by the actual bed temperature close to the cooling surface.

Eq. (1-27) may be written in the following form:

$$h_{rad} = \sigma\, e_{eff}\,(T_b^2 + T_w^2)\,(T_b + T_w) \qquad \text{Eq. (1-28)}$$

For the temperatures of interest, this is closely approximated by the following relation:

$$h_{rad} = 4\,\sigma\, e_{eff}\,(T_m^3) \qquad \text{Eq. (1-29)}$$

Where

$$T_m = (T_b + T_w)/2$$

The bed may be considered to be an impervious cylinder surrounding the cooling tube. e_{eff} is then:

$$e_{eff} = \frac{1}{1/e_w + \left(A_w / A_b\right)\left(1/e_b - 1\right)} \qquad \text{Eq. (1-30)}$$

A_w is the surface area of tube and A_b the surface area of bed surrounding it. As the particles are very close to the tube,
$A_w \approx A_b$. Then:

$$e_{eff} = \frac{1}{1/e_w + 1/e_b - 1} \qquad \text{Eq. (1-31)}$$

According to Zabrodsky [1.28], $e_b \approx 1$ and thus $e_{eff} \approx e_w$. However, experimental studies have indicated much lower values of bed emissivity. Grace [1.1] examined the data reported by Botterill [1.17] and suggested the following formula:

$$e_b = 0.5\,(1 + e_s) \qquad \text{Eq. (1-32)}$$

where e_s is the emissivity of solid particles. The experimental data included $D_s = 0.25$ to 1.5 mm, $u/u_{mf} = 1.2$ to 4.0, bed temperatures 450 to 1450 °C, and particle emissivities from 0.23 to 0.6. Data on solid particle emissivity may be found in Section 412.5 of the *Fluid Flow* volume and other books dealing with radiation heat transfer. The emissivity of a particle is generally the same as that of a larger body of the same material.

Martin [1.30], whose analytical model for conduction and convection heat transfer has been presented earlier, calculated radiation heat transfer by Eq. (1-29) with $e_{eff} = 0.5$. The high temperature data analyzed were from several sources, including [1.56]. Bed temperatures were up to 960 °C. Particles included sand, coke, and quartz sand, with diameters from 0.1 to 1.39 mm. Good agreement is reported with data for maximum heat transfer as well as that over a range of velocities.

Baskakov [1.5] has approximated Eq. (1-27) by the following formula for $0.3 < e_s < 0.6$:

$$h_{rad} = 7.3\,\sigma\, e_s\, e_w\, T_w^3 \qquad \text{Eq. (1-33)}$$

Note that $T_b > T_w$. Yamada *et al* [1.37] applied this formula to their data for alumina particles 0.183 and 0.352 mm diameter. Good agreement was found using e_s and e_w as 0.8 and 0.9 respectively. As $e_s = 0.8$ is well beyond the recommended range of Eq. (1-33), the value of this confirmation is questionable. Yamada *et al* also report satisfactory agreement with the semi-theoretical correlation of Flamant and Menigault [1.32] when the adjustable factors were obtained from their own data.

B. RESULTS OF VARIOUS STUDIES

Many theoretical and experimental studies have been conducted on radiant heat transfer between beds and heat transfer surfaces. The results of all studies agree that the contribution due to radiation is negligible at low temperatures but is significant at higher temperatures. However, there is a lack of consensus regarding the temperature at which radiation becomes significant and its magnitude. The conclusions of each researcher are supported by some experimental data.

Many of the studies on radiation heat transfer have been reviewed by Saxena and Gabor [1.52], and Grewal [1.7], among others. The results of some of the studies are discussed here.

Szekely and Fisher [1.53] analyzed a single particle model and concluded that radiation heat transfer is insignificant at temperatures less than 1000 °C. Yoshida *et al* [1.54], based on their theoretical analyses and experiments, concluded that radiation contribution is insignificant up to 1200 °C. The thermal analysis of Vedmurthy and Shastri [1.55] showed that for particles of 0.5 to 3 mm diameter in the temperature range 800 to 1100 °C, radiation contributes 17 to 30% of the total heat transfer. Kharachenko and Makhorin [1.56] carried out tests at temperatures up to 1050 °C. They found that the maximum bed-to-surface heat transfer coefficient varied linearly with the bed to surface difference in temperatures. They concluded that radiation heat transfer was insignificant, as otherwise a non-linear relation would have been found.

Kolar *et al* [1.57] used an alternate slab model to calculate heat transfer for various particle sizes and temperatures. Contribution due to radiation was found to increase with particle diameter. In one of their calculations, with T_B = 900 K and T_W = 800 K, contribution due to radiation increased from 20% to 44% as the size of dolomite particles increased from 0.25 to 1.0 mm.

The conclusion of Zabrodsky *et al* [1.13] is that radiation can be neglected at temperatures up to 1000 °C. According to Gelperin and Einstein [1.21], this limit is about 900 °C.

Botterill and Sealey [1.58] and Botterill [1.17] have reviewed some experimental work done by Russian researchers. The radiation component was found to vary from 5-10% at 500 °C to 50-60% at 1400 °C. Radiation flux was found to be independent of fluidizing velocity.

Shah [1.40] analyzed the data of Kharachenko and Makhorin [1.56] for quartz and Chamotte particles (D_p = 0.34 to 1.6 mm) at temperatures from 500 to 900 °C. The maximum heat transfer coefficients showed satisfactory agreement with his correlation, Eqns. (1-14) and (1-15).

Shah also compared his correlation with the data of Ku *et al* [1.14] for D_p = 1.08 to 1.2 mm at temperatures up to 815 °C. Satisfactory agreement was found.

Grewal and Saxena [1.22] compared their correlation, Eq. (1-18), the data of Tischenko and Khvastukin [1.50] for a 36 mm diameter tube in air fluidized beds of high alumina (D_p = 0.97 to 1.45 mm) at temperatures up to 1100 °C. Satisfactory agreement was found, apparently without any correction for radiation heat transfer.

Renzhang *et al* [1.59] conducted tests on a single horizontal tube. For 0.802 mm diameter particles, they found radiation contribution to be 3, 24, and 27 percent at bed temperatures of 600, 950, and 1000 °C, respectively. For D_s = 0.497 mm, radiation contribution was 17% at T_b = 950 °C.

C. CONCLUSION AND DESIGN RECOMMENDATIONS

There is considerable quantitative disagreement between the results of various analytical methods and insufficient evidence to recommend any one of them exclusively. As what matters for practical purpose is the prediction of total heat transfer, it is advisable to use the method for radiation heat transfer, which has been shown to be successful with the method used for calculating conduction-convection contribution. The following recommendations are therefore made:

1. When Martin's method, Eqns. (1-4) to (1-11), is used to calculate conduction-convection contribution, calculate h_{rad} by Eq. (1-29) with e_{eff} = 0.5.

2. When the conduction-convection component is calculated by the correlation of Shah, Eqns. (1-14) and (1-15), radiation heat transfer is neglected up to 900 °C bed temperatures. At higher temperatures, estimate e_{eff} from Eqns. (1-31) and (1-32).